Different Representations of the Earth Impedance Matching in Distance Protection Relays or
What Impedance Does a Digital Distance Protection Relay Measure?

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Introduction

Digital distance protection relays use different algorithms for calculating the line impedance to the fault location from the current and voltage measurements. The calculated impedances differ under certain circumstances as shown in the following example.

A single-phase L1-ground fault is simulated in three different distance protection relays (of type A, type B, and type C). Each relay measures the following voltage and current values:

\[ U_{L1E} = 20 \text{ V } e^{0^\circ}, \]
\[ I_{L1} = I_E = 2 \text{ A } e^{-30^\circ}. \]

The line impedances calculated by the respective distance protection relays are as follows:

Type A: \[ X = 4.4 \text{ Ohm}; R = 7.0 \text{ Ohm}, \]
Type B: \[ X = 4.2 \text{ Ohm}; R = 6.2 \text{ Ohm}, \]
Type C: \[ X = 4.2 \text{ Ohm}; R = 8.2 \text{ Ohm}. \]

The different calculated impedance values are caused in particular by the different representations of the earth impedance matching. This paper focuses on the underlying algorithms and their impact on testing the digital distance protection relays.

Basics

A distance protection relay measures impedance to locate the short-circuit point. Figure 1 shows the equivalent circuit diagram of a doubly fed line with a single-phase fault in phase L1. The digital distance protection relay measures the voltage and current values at the point M and calculates the impedance using the Kirchhoff's loop rule equation

\[ U_{L1E} = I_{L1} \cdot Z_L - I_E \cdot Z_E + R_F \cdot I_F. \]

In the above equation, the term \( R_F \cdot I_F \) due to the arc is unknown. In particular, the current \( I_F \) consists of the short-circuit currents from the left and the right line and, consequently, gives rise to an unknown voltage over \( R_F \). Hence, the distance protection algorithms have to use simplifying assumptions to solve the Kirchhoff's loop rule equation. On the other hand, the term \( I_E \cdot Z_E \) is resolved using the so-called earth impedance matching.

![Figure 1: Single-phase fault in a doubly fed line](image)

**M:** Measurement point

**\( Z_L \):** Line impedance from M to the fault location

**\( Z_E \):** Grounding impedance from M to the fault location

**\( R_F \):** Arc resistance

**\( Z_{L2} \):** Line impedance from the fault location to the end of the line

**\( Z_{E2} \):** Grounding impedance from the fault location to the end of the line

**\( Z_{VX} \):** Impedances of the preceding network
Earth Impedance Matching Factors

The earth impedance matching factors (K factors) are line parameters (independent of the fault location) describing the ratio of the line and grounding impedances. The K factors are used for calculating the line impedance in the ground-fault loop by allowing for the grounding impedance effect. In the equivalent circuit diagram (see Figure 2), the ground-fault loop impedance $Z_s$ is split into the line impedance $Z_L$ and the grounding impedance $Z_E$. In case of a dead short circuit ($R_F = 0$), the following equations apply:

$$Z_s = Z_L + Z_E$$
$$Z_L = R_L + jX_L$$
$$Z_E = R_E + jX_E$$

**Figure 2: Line section with an L1-ground short circuit**

The following K factors definitions are commonly used:

The complex ratio

$$K_L = \frac{Z_E}{Z_L} \quad (Eq. 1)$$

is a line constant expressing the ratio of the grounding and line impedances. Another common definition is

$$K_0 = \frac{Z_0}{Z_1} \quad (Eq. 2)$$

where $Z_0$ is the neutral line impedance and $Z_1$ is the positive-sequence system impedance. The factors $K_L$ and $K_0$ can be converted using the relation $K_0 = (K_L - 1)/3$.

Alternatively, the earth impedance matching factor is represented by a couple of real values:

$$K_r = \frac{R_E}{R_L} \quad K_x = \frac{X_E}{X_L} \quad (Eq. 3)$$

While the factors $K_L$ and $K_0$ can be converted directly, for the conversion between $K_r$ and the couple of values $(K_r, K_x)$ another line parameter, the ratio $X_L/R_L$, is necessary. Then the transformation formulas are as follows:

$$K_L = K_r(1 + jK_rX_L) + K_x(1 - jX_L/R_L) \quad (Eq. 4)$$

The ratio $X_L/R_L$ is the tangens of the line angle $\varphi_L$.

Consequently, the conversion of the different K factors depends on the line angle. This is really why the distance protection algorithms for impedance calculation using different K factors may provide different results.

**Distance Protection Algorithms**

In the following derivations, we assume a single-phase fault in phase $L_x$.

**Distance protection algorithm of type A "complex $K_L$"**

The algorithm of type A uses for calculating the line impedance in the ground-fault loop the complex factor $K_L$ as follows:

$$U_{LxE} = I_{Lx}Z_L - I_EZ_E \quad (Eq. 5)$$

The substitution $Z_E = K_LZ_L$ yields:

$$U_{LxE} = I_{Lx}Z_L - I_EK_LZ_L \quad (Eq. 6)$$

Assuming $I = I_{Lx} = -I_E = I_F$ to simplify the derivation and using the notation $U = U_{LxE}$, where $U$ and $I$ are the voltage and current values respectively in the ground-fault loop measured by the relay, Eq. 6 yields:

$$Z_{TypeA} = R_{TypeA} + jX_{TypeA} = \frac{U/I}{1 + K_L} \quad (Eq. 7)$$

where $Z_{TypeA}$ is the line impedance $Z_L$ calculated by the algorithm A.

**Distance protection algorithm of type B "RE/RL, XE/XL"**

The distance protection relays of type B use for calculating the line impedance in the ground-fault loop the couple of real values $K_r$ and $K_x$ as follows:

$$U_{LxE} = I_{Lx}(R_L + jX_L) - I_E(R_E + jX_E) \quad (Eq. 8)$$

The substitutions $R_E = K_rR_L$ and $X_E = K_xX_L$ yield:

$$U_{LxE} = I_{Lx}(R_L + jX_L) - I_E(K_rR_L + jK_xX_L) \quad (Eq. 9)$$

Assuming $I = I_{Lx} = -I_E = I_F$ and using the notation $U = U_{LxE}$, the imaginary and real parts of the line impedance calculated by the algorithm B are as follows:

$$X_{TypeB} = \frac{\text{Im}\{U/I\}}{1 + K_r} \quad (Eq. 10)$$
$$R_{TypeB} = \frac{\text{Re}\{U/I\}}{1 + K_r}$$
Distance protection algorithm of type C "arc separated"

The distance protection relays of type C use for calculating $X_L$ the same algorithm as the relays of type B. Different is, however, the calculation of the real part of the line impedance. This approach takes the arc resistance $R_F$ into consideration assuming the current $I_F$ equal to the line current $I_L$.

The underlying idea here is to put together the arc resistance and the line resistance. In other words, the real part of the grounding impedance is subtracted from the real part of the ground-fault loop impedance ($U/I$). The grounding resistance can then be resolved from the calculated value $X_L$ as a function of the factor $K_r$ and the line angle. The imaginary and real parts of the line impedance are then as follows:

$$X_{Type C} = \frac{\text{Im}\{U/I\}}{1 + K_r}$$
$$R_{Type C} = \text{Re}\{U/I\} - X_{Type C} \cdot \arctan(\phi_L) \cdot K_r$$

**Table 2: Interpretation of the distance protection algorithms**

<table>
<thead>
<tr>
<th>Type</th>
<th>$X_{meas}$</th>
<th>$R_{meas}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$X_L + \text{Im}{R_F/(1 + K_L)}$</td>
<td>$R_L + \text{Re}{R_F/(1 + K_L)}$</td>
</tr>
<tr>
<td>B</td>
<td>$X_L$</td>
<td>$R_L + R_F/(1 + K_r)$</td>
</tr>
<tr>
<td>C</td>
<td>$X_L$</td>
<td>$R_L + R_F$</td>
</tr>
</tbody>
</table>

Table 2 presents the key result of the analysis. The impedance $Z_{meas} = R_{meas} + j X_{meas}$ measured by the digital distance protection relays consists of the line impedance $Z_L$ and a part due to the arc. The amount of the arc resistance $R_F$ added to the line impedance depends on the distance protection algorithm.

**Figure 3: Vector diagram for type A relays "complex $K_L$"**

In the relays of type A, the earth impedance matching factor can be set complex. In this case the part due to the arc is also complex.

**Figure 4: Vector diagram for type B relays "RE/RL, XE/XL"**

The relays of type B reduce the arc resistance by the real factor $1/(1 + K_r)$. 

The term $U/I$ is the ground-fault loop impedance $Z_s$ consisting of $Z_L$, $R_F$ and $Z_E$. A comparison with the equivalent circuit diagram shown in Figure 2 allows the following interpretation of the formulas:
The relays of type C measure the impedance without compensating for the arc resistance. The distance protection relays of type A, B and C differ in the definition of the measured impedance. Consequently:
- The scaling in the impedance plane changes.
- Figure 3, 4 and 5 show the respective axis labeling.
- The impedances measured by different relays are not comparable in the same impedance plane.
- All distance protection algorithms provide the same results (R_A and X_A) at the line angle (R_C = 0).
- Outside the line angle, the arc resistance adds up to the line impedance depending on the position in the impedance plane.

All distance protection algorithms discussed work correctly. However, when setting the zones and testing, take into consideration the different underlying representations of the earth impedance matching!

Considering the Earth Impedance Matching in Testing

In the test software Test Universe 2.0 all the above distance protection algorithms are implemented. The Distance protection parameters dialog window (see Figure 6) allows independent entering the earth impedance matching factors according to Eqs. 1, 2, or 3 and selecting the algorithm for calculating the voltage and current values. The test software and the relay must use the same algorithms. Using different algorithms in the test software and in the relay leads to incorrect test results.

Table 3: Setting the algorithm in Test Universe 2.0

<table>
<thead>
<tr>
<th>Type</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>&quot;complex K_L&quot;</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Calc. with RE/RL and XE/XL" /></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Separate arc resistance" /></td>
</tr>
<tr>
<td>Type B</td>
<td>&quot;RE/RL, XE/XL&quot;</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Calc. with RE/RL and XE/XL" /></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Separate arc resistance" /></td>
</tr>
<tr>
<td>Type C</td>
<td>&quot;arc separated&quot;</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Calc. with RE/RL and XE/XL" /></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Separate arc resistance" /></td>
</tr>
</tbody>
</table>

In Test Universe Version 1.6 and earlier, the algorithm used by the relay was coupled with setting the earth impedance matching factors. Recently, distance protection relays are coming on the market allowing the user to select how the K factors are entered without affecting the working algorithm. Consequently, a distance protection relay can use e.g. the algorithm of type B "R_A/R_C, X_A/X_C", although the complex K_L is entered in the relay. In Test Universe Version 1.6, the art how the earth impedance matching factors are entered must correspond with the working algorithm to arrive at correct results.
Where is the Information on Settings Available?

For the algorithms used by the distance protection relays refer to the relevant documentation. Since the algorithms are not always published, some typical examples are listed below:

Table 4: Examples of distance protection relays
(no liability assumed)

<table>
<thead>
<tr>
<th>Type</th>
<th>Relay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>&quot;complex KL&quot; REL 316,</td>
</tr>
<tr>
<td></td>
<td>Relay with MHO characteristic</td>
</tr>
<tr>
<td>Type B</td>
<td>&quot;RE/RL, XE/XL&quot; 7SA5**, 7SA6**</td>
</tr>
<tr>
<td>Type C</td>
<td>&quot;arc separated&quot; SEL321, SEL421, REL4**, Rel5**, LFZR, EPAC, MICOM (Valid for polygonal zones only! For MHO characteristics applies type A!)</td>
</tr>
</tbody>
</table>

If in doubt, you can find out the algorithm used in the relay by testing the relay using the QuickCMC test module by means of the example in Section "Introduction" as follows:

The parameters of the protected line for 100 percent of the line length are:

- \( X_{L100\%} = 10 \text{ Ohm} \)
- \( R_{L100\%} = 2.5 \text{ Ohm} \)
- \( X_{E100\%} = 5 \text{ Ohm} \)
- \( R_{E100\%} = 1 \text{ Ohm} \)

The corresponding settings of the QuickCMC module are:

- Line angle: 75.96°
- \( K_r = R_{RE/RL} = 0.4 \)
- \( K_x = X_{XE/XL} = 0.2 \)
- \( K_L = 0.217 e^{-12.53°} \)

1. Set the QuickCMC module as above.
2. Set the same parameters in the relay.
3. Set a zone in the relay between 6.2 and 7.0 Ohm.
4. Test the relay. If the relay trips, it is of type B.
5. If the relay does not trip, set a zone between 7.0 and 8.2 Ohm.
6. Test the relay. If the relay trips, it is of type A.
7. If the relay does not trip, it is of type C.

Summary

Distance protection relays of different types define the measured impedance differently. The differences are due to the representation of the earth impedance matching and the corresponding compensation of the arc resistance.

The definition of the measured impedance must be considered both in the zone setting and in testing. Test Universe allows testing of distance protection relays of all the above types. The relays are tested easily using their own earth impedance matching.

Literature

[1] SEL2RIO Converter A. Dirks 1999
[4] Digitaler Abzweigschutz 7SA513 V3.2 – Benutzerhandbuch, SIEMENS